International Conference on Mathematical Physics "Kezenoi-Am 2016"

Institute of Mathematical Physics and Seismodynamics
Chechen State University

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Welcome Address

Welcome to the International Conference on Mathematical Physics "Kezenoi-Am 2016"! This a preliminary version of the conference abstracts book.

Plenary Speakers Abstracts

Analytic formulae for charge two monopole fields

Victor Enolski
National University "Kyiv Mohyla Academy", Kyiv, Ukraine

The ADHMN-construction (Atiyah-Drinfeld-Hitchin-Manin-Nahm) of the Higgs and gauge fields for a nonabelian monopole leads to solving of a Weyl equation, that is a linear ODE with "potentials" given by the so called Nahm data. Even in the case of charge two, where the Nahm data are expressible in terms of elliptic functions, the analytic expressions for the monopole fields are still unknown. We overcome the problem using as we call it *Lesser known Nahm Ansatz* comparatively to the well known Nahm Ansatz which reduces ADHM instanton construction to static, ADHMN monopole case. We report complete analytic description in $\mathbb{R}^3$ of charge two monopole fields in terms of four solutions of a quartic (Atiyah-Ward constraint) and four transcendents, given as incomplete second kind elliptic integrals depending in these solutions. We also present analytic expression for the energy density and its visualisation in picture form.

Parabolic equation of normal type connected with 3D Helmholtz system

Andrey Fursikov
Moscow State University, Russia

The talk will be devoted to the normal parabolic equation (NPE) connected with $3D$ Helmholtz system whose nonlinear term $B(v)$ is orthogonal projection of nonlinear term for Helmholtz system on the ray generated by vector v. To study NPE of such kind is interesting by the following reasons:

- Its study open the way to construct the method of nonlocal stabilization by feedback control for $3D$ Helmholtz as well as for $3D$ Navier-Stokes equations.

- Its study can help to understand better difficulties that one should overcome to solve Millennium problem on non-local existence of smooth solution for $3D$ Navier-Stokes equations.

The structure of dynamical flow corresponding to this NPE will be described. Besides, the non local stabilization problem for NPE by starting control supported on arbitrary fixed sub-domain will be formulated. The main steps of solution to this problem will be discussed and connection of this problem with non-local stabilization problem for $3D$ Helmholtz system will be explained.

Braided Yangians

Dmitry Gurevich

Valenciennes University, France

It is well known that the Yangian $Y(gl_m)$, introduced by V. Drinfeld, and associated with the Yang quantum R-matrix, plays the role of a symmetry group for Non-linear Schrodinger model. It is also closely related to W-algebras. It has many interesting properties. However, its q-analog, called q-Yangian, and defined as a "half" of a quantum affine group, is not properly studied. We suggest a new candidate to the role of the q-analog of the Yangian $Y(gl_m)$. We call it "braided Yangian". Its properties are more similar to these of the usual Yangian. Also, we associate "braided Yangians" with other quantum R-matrices, arising from Hecke symmetries (i.e. braidings of Hecke type) via the Yang-Baxterization procedure. We construct a rich representation theory of these braided Yangians by using an analog of the evaluation morphism and discuss "braided bosonization" of these objects. We compare this bozonization with approach based on the Zamolodchikov-Faddeev algebra.

Integrable magnetic geodesic flows on 2-torus: new example via quasi-linear system of PDEs

Andrey Mironov

Novosibirsk State University, Russia

The only one example has been known of magnetic geodesic flow on the 2-torus which has a polynomial in momenta integral independent of the Hamiltonian. In this example the integral is linear in momenta and corresponds to a one parametric group preserving the Lagrangian function of the magnetic flow. We consider the problem of integrability on one energy level. This problem can be reduced to a remarkable Semi-hamiltonian system of quasilinear PDEs and to the question of existence of smooth periodic solutions for this system. Our main result states that the pair of Liouville metric with zero magnetic field on the 2-torus can be analytically deformed to a Riemannian metric with small magnetic field so that the magnetic geodesic flow on an energy level is integrable by means of a quadratic in momenta integral. Thus our construction gives a new example of smooth periodic solution to the Semi-hamiltonian quasilinear system of PDEs. The talk is based on the joint paper with Misha Bialy (Tel-Aviv) and Sergey Agapov (Novosibirsk).

Autoresonance in a model of terahertz wave generator

Victor Novokshenov

Institute of Mathematics, Ufa Science Centre of the RAS, Russia

A model of generator of electromagnetic waves is studied which is based on the stack of Josephson junctions. The model is described by a chain of coupled sine-Gordon equations on phases of electromagnetic field under small dissipation and constant current pumping. We establish the conditions of resonant excitation of the field under various parameters of the system. The auto-resonant nature of Josephson radiation is revealed in the dependence of frequency on the pumping level.

Differential operators, geodesics and dynamics of localized quantum states on singular spaces

Andrey Shafarevich

Moscow State University, Russia

We discuss properties of evolution equations - for example, Schroedinger or wave equations - on singular spaces. These spaces are obtained from graphs by replacing vertices by small dimensional Riemannian manifolds. Behavior of localized solutions appear to be connected with global properties of geodesic flows on manifolds and with popular problems of analytic number theory.

Polynomial forms for quantum Calogero-Moser Hamiltonians and commutative sub-algebras in universal enveloping algebras

Vladimir Sokolov

Landau Institute for Theoretical Physics, Russia

We find transformations that bring the quantum Calogero-Moser Hamiltonians for $n = 2, 3, 4$ to differential operators with polynomial coefficients. These operators are related to special commutative sub-algebras in the universal enveloping algebras of $gl(n + 1)$.

Bäcklund transformations and Abel equations

Andrey Tsiganov

Institute of Physics, St.Petersburg University, Russia

Auto Bäcklund transformation of Hamilton-Jacobi equations is a very particular time-independent transformation of variables, which conserves not only the Hamiltonian character of the equations of motion, but also the Hamilton-Jacobi equations. These canonical transformations preserving the algebraic form of Hamiltonians can be applied to:

- discretization and numerical solution of initial equations of motion;

- construction and investigation of integrable multivalued algebraic maps;
- construction of new integrable systems using hetero Bäcklund transformations;
- classification of the Poisson brackets compatible with the canonical bracket.

If integration by quadratures of the given integrable Hamiltonian systems can be reduced to inversion of Abelian sums of holomorphic hyperelliptic integrals, then an existence of auto Bäcklund transformation is equivalent to an existence of the Abel differential equations. It allows us to use all the machinery developed by Euler, Abel, Jacobi, Weierstrass, Richelot et al to construct. auto Bäcklund transformations for classical integrable systems. We want to discuss an algebraic construction of the auto Bäcklund transformations for the Hamilton-Jacobi equations solvable by inversion of the Abel quadratures on the hyperelliptic and non-hyperelliptic curves. The main difference between Bäcklund transformations related to hyperelliptic and non-hyperelliptic curves is a valency of the corresponding canonical transformation of variables.

Toric topology and geometry

Taras Panov
Moscow State University

Toric topology is a new area of mathematics that emerged at the end of the 1990s on the border of equivariant topology, algebraic and symplectic geometry, combinatorics, and commutative algebra.

The key players in toric topology are moment-angle manifolds, a class of manifolds with torus actions defined in combinatorial terms. Construction of moment-angle manifolds relates to combinatorial geometry and algebraic geometry of toric varieties via the notion of a quasitoric manifold. Discovery of remarkable geometric structures on moment-angle manifolds led to important connections with classical and modern areas of symplectic, Lagrangian, and non-Kaehler complex geometry. A related categorical construction of moment-angle complexes and polyhedral products provides for a universal framework for many fundamental constructions of homotopical

topology. The study of polyhedral products is now evolving into a separate subject of homotopy theory. A new perspective on torus actions has also contributed to the development of classical areas of algebraic topology, such as complex cobordism.

After an introductory part describing the construction and the topology of moment-angle complexes, we shall concentrate on several interesting geometric properties of moment-angle manifolds, emphasising complex-analytic, symplectic and Lagrangian aspects.

The talk is based on joint works with Victor Buchstaber, Andrei Mironov, Yuri Ustinovsky and Mikhail Verbitsky.

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